TCPS Math Enrichment

Days Thursdays
Time 3:10pm-4:10pm
Instructor Jeff Candy
Goal Develop superhuman ability at mathematical problem solving!

Q: The letters G, A, U, S, and S are written on five tiles, one letter per tile. If Amy selects two tiles at random, what is the probability she gets two Ss?

$$(A)\ \frac{3}{5} \quad (B)\ \frac{2}{5} \quad (C)\ \frac{1}{8} \quad (D)\ \frac{1}{10} \quad (E)\ \frac{1}{20}$$

Does solving this problem sound like as much fun as getting braces? Well, what if by the end of the school year you'd be able to solve problems like this in a *snap*, as easily as multiplying $\frac{2}{5} \times \frac{1}{4}$. Would that interest you?

Seriously, let's back up a bit. Consider the following reflection, by the famous French mathematician Laplace, on the *invention* of the base-10 number system:

> It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it has lent to computations put our arithmetic in the first rank of useful inventions; and we shall appreciate the grandeur of the achievement the more when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity.
>
> *Pierre-Simon Laplace (1749-1827)*

This quote from Laplace shows that even a great mind like Archmides lived his whole life without ever imagining something *really simple* – so simple that *everyone* considers it almost obvious: the use of a base (base 10 in particular) in expressing numbers. Had you lived in ancient Rome, you would have written the number 256 this way:

$$256 = \mathrm{CCLVI}_{\mathrm{Roman}}\,. \tag{1}$$

In fact there are all sorts of ways to write the same thing (after all, the weight and volume of 256 apples doesn't change when they're called CCLVI apples). More generally, the number 256 can be written in base 2 (binary), 3, 4, 8 (octal) and 16 (hexidecimal) as

$$256 = 100000000_2 \tag{2}$$
$$= 100111_3 \tag{3}$$
$$= 2011_4 \tag{4}$$
$$= 400_8 \tag{5}$$
$$= 100_{16}\,. \tag{6}$$

Above, the subscript under the number indicates the *base*. So, the deeper message here is that even something as intuitive and "easy" as expressing the number 256 can become complicated when using an *unfamiliar technique*. And so it goes for much of mathematics. What seems complicated and useless at first sight may turn out to be elegant and simple if you just give it a chance.

The goal of this enrichment class is two-fold:

1. Its an opportunity for students with a fondness for mathematics to be exposed to concepts and techniques not normally introduced before high school.

2. Its a chance to *train* for math competition. Mostly, this means just doing alot of lot of problems! In some sense math competition is like sports – lots of daily training that'll pay off at competition time.

But you don't have to compete. That's optional. The only true requirements for attendance are a *desire to learn* and a *willingness to put pencil to paper*.

Hope to see you there!